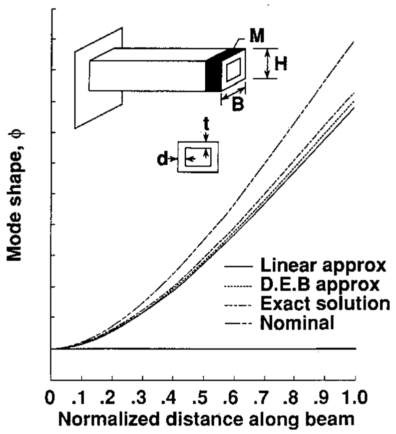

Fig. 3 Comparison of DEB and linear approximations of first bending mode shape for 50% perturbation of beam width *B*. [Nominal values; $H = 5.0$ in. (12.7 cm), $M = 0$ lbm, $B = 3.75$ in. (9.53 cm).]

Figure 1 shows a graph of the fundamental eigenvalue as a function of the height of the beam H for values perturbed from the nominal value (H_o) of 5.0 in. (12.7 cm). For as much as a 50% increase in H, the new approximation is within 2% of the exact solution, compared to 12% for the Taylor series approximation. It is also evident from the figure that, for decreases in H exceeding 45%, the Taylor series method gives negative values for the eigenvalues.

Figure 2 illustrates the application of the DEB method to simultaneous changes in three design variables: tip mass, bending inertia, and cross-sectional area of the beam. The nominal values of the design variables (corresponding to $\theta = 0$) are $M = 5.0$ lbs (2.268 kg), $I = 28.863$ in.^{4} (1201.4 cm^4), and $A = 7.0$ in.^{2} (45.2 cm^2). Perturbations were made in increments of 10% of these nominal values. Results from the DEB approximation were very encouraging, and for even as much as a 50% increase in the design variables there was still only a 32% error compared to nearly a 100% error for the Taylor series approximation. Similar trends occurred for other variables in approximating frequencies of the first and second bending modes.

Figure 3 shows a sketch of the first bending mode shape for the nominal design, the Taylor series approximation, the DEB approximation, and the exact solution for a 50% perturbation in the beam width B. Both approximations are very accurate, but the DEB approximation is closer to the exact curve.

Concluding Remarks

This note has demonstrated that sensitivity equations, when interpreted as differential equations, may be used to generate accurate approximations. To date, the method has been developed and demonstrated for frequency and mode shape approximations. In principle, the method is applicable to approximating any quantity for which an analytical sensitivity formula is available. For example, approximating displacements is a potential extension of this concept.

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Three-Dimensional Shape Optimization with Substructuring

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Introduction

PREVIOUS research on shape optimization has been focused on design sensitivity theory,[1,2] and design modeling techniques,[2–7] with usually the entire structure being analyzed and subsequently optimized. In a real design environment, one often encounters the fact that only a small part of a complicated component is allowed to change. In this case, substructure analysis is a more efficient approach than the full-structure finite element approach in an iterative design process. Since, especially in structural optimization, the structure has to be analyzed for each design iteration, the substructure approach is a very efficient tool for a large problem.[8] In shape optimization, the computational advantage is not only in analysis but also in the sensitivity calculations. The CPU reduction in analysis and the sensitivity calculations result in a more efficient optimization process.

Substructure Analysis

The substructure concept is, first, to divide the complete structure into several regions, condense the interior degrees of freedom to the boundaries of the regions, and then establish

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the relationship between the interfaces of the substructures. During the static condensation process, the interior degrees of freedom are expressed in terms of the exterior degrees of freedom, and only the exterior degrees of freedom, which are usually very small compared to the complete structure, have to be solved.

Assuming an nth substructure, the equilibrium equation can be written as follows:

$$\begin{Bmatrix} K_{ii}^n & K_{ie}^n \\ K_{ei}^n & K_{ee}^n \end{Bmatrix} \begin{Bmatrix} z_i^n \\ z_e^n \end{Bmatrix} = \begin{Bmatrix} f_i^n \\ f_e^n \end{Bmatrix} \tag{1}$$

where i and e indicate the interior and exterior degrees of freedom, respectively. The interior and exterior degrees of freedom can be separated as

$$z_i^n = -\left(K_{ii}^n\right)^{-1} K_{ie}^n z_e^n + \left(K_{ii}^n\right)^{-1} f_i^n \tag{2}$$

$$z_e^n = -\left(K_{ee}^n\right)^{-1} K_{ei}^n z_i^n + \left(K_{ee}^n\right)^{-1} f_e^n \tag{3}$$

Substituting Eq. (2) and Eq. (3), the interior degrees of freedom can be removed as

$$\left[K_{ee}^n - K_{ei}^n\left(K_{ii}^n\right)^{-1} K_{ie}^n\right] z_e^n = f_e^n - K_{ei}^n\left(K_{ii}^n\right)^{-1} f_i^n \tag{4}$$

Notice that in Eq. (4) the interior degrees of freedom z_i^n are condensed to the exterior degrees of freedom z_e^n. The condensation process is applied to each substructure. The final equilibrium equation can be obtained by assembling all the exterior degrees of freedom of each substructure. The reduction in problem size for the system of equations may result in a computational advantage over the full-structure approach. However, one also has to pay the penalty for matrix inversion of K_{ii}^n and matrix multiplication in Eq. (4). In a one-time-only analysis, the substructure formulation may not have any advantage over the full-structure approach, but in an iterative design environment, as in structural optimization, a significant computational savings can be achieved by the substructure formulation in which only the affected and changed substructures need be updated. In other words, only the stiffness matrix for the modified part is reformulated and reduced to the exterior degrees of freedom for the changed design; the unchanged portions are kept the same.

Design Sensitivity Analysis

In addition to the computational advantage over the full-structure finite element analysis, the substructure approach is more efficient in shape sensitivity calculations. In Refs. 9 and 10, a hybrid approach for shape design sensitivity analysis was proposed. In this approach, both the adjoint variable method and the direct method were used for shape sensitivity analysis. Both need integration over the same physical domain as the formulation of the stiffness matrix, and both require a velocity term, which is the design change or perturbation that exists only in the modified portion of the part and vanishes when the geometry is kept fixed. Therefore, the contribution to those quantities is only from the portion that is subject to change. Obviously, if the integration is carried out over the elements of the changed portion only, a significant computational advantage can be achieved.

The approach is easy to implement. First of all, one defines the major structure that is kept fixed as one substructure and the minor structure that is subject to change as the residual structure. Before the optimization, the stiffness matrix of the major substructure is formulated and reduced to the exterior degrees of freedom and stored in a data base for later use. In the optimization process, only the stiffness matrix for the residual structure has to be generated. Since the residual structure is usually small, the formation of the new stiffness matrix is less expensive than in the full-structure finite element approach, where the stiffness matrix of the entire structure has to be reformulated and decomposed. In the design sensitivity calculation, the elements in the residual structure are identified and integrated over to obtain the necessary sensitivities. Since the domain integration is very time-consuming, as in the stiffness matrix formulation, the savings may be significant.

Design Example

A front steering knuckle, shown in Fig. 1, is used as an example. The finite element model contains 260 quadratic hexahedral elements, 2640 grid points, and more than 7500 degrees of freedom. The loading cases considered are forward panic braking, cornering, curb impact, rim pull, and 8G vertical load. The design goal is to minimize the use of material and still meet the design criteria.

Assume that only the shape of the upper control arm attachment, shown as the shaded portion of Fig. 1, is allowed to change, due to other requirements. In this case, one may model the upper control arm attachment as the residual structure, and the rest as substructure one. Note that the design model that associates the design variables to the physical geometry is generated on the residual structure only. Six design variables are defined to change the thickness of the control arm, and a cubic polynomial surface is then defined for the upper surface.[6]

The initial knuckle design is first used to test the superelement formulation for the adjoint variable and the direct method. Since there are four stress constraints active in this design (due to curb impact load), four reanalyses are required for the adjoint variable approach. For the direct approach, six

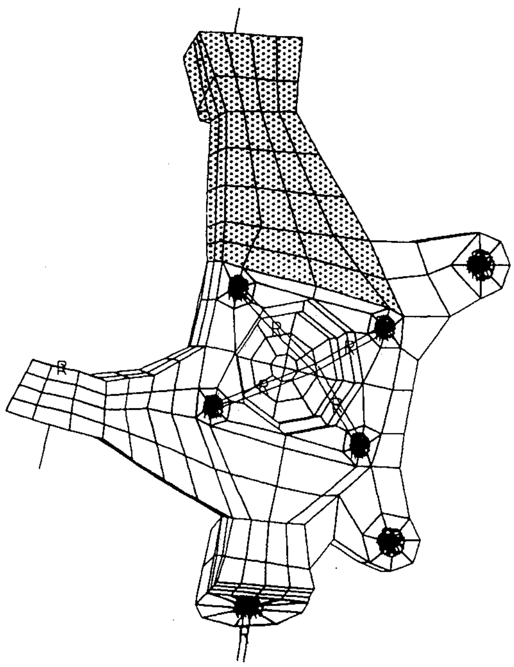

Fig. 1 Front knuckle model.

Table 1 Comparison of CPU seconds on IBM 3090, adjoint method

Step	Function	Full-structure	Substructure
1	Stress analysis	421.55	197.27
2	Problem definition	11.47	6.47
3	Reanalysis	19.90	35.28
4	Sensitivity & optimization	103.73	16.88
Total		556.65	255.90

Table 2 Comparison of CPU seconds on IBM 3090, direct method

Step	Function	Full-structure	Substructure
1	Stress analysis	426.71	200.51
2	Problem definition	98.76	20.40
3	Reanalysis	30.68	27.16
4	Sensitivity & optimization	39.37	6.92
Total		595.52	254.99

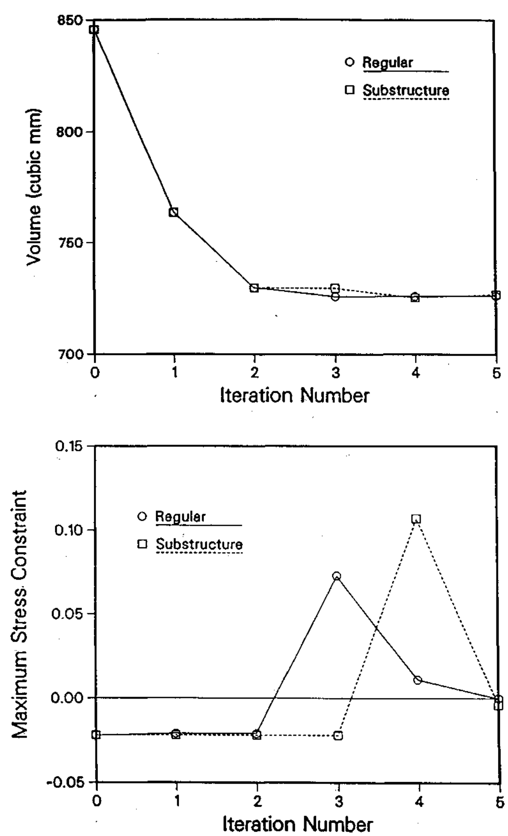

Fig. 2 Design histories of front knuckle.

reanalyses are needed because six design variables are defined. The CPU times are quite different between the full-structure approach and the superelement formulation.

Table 1 shows the CPU time comparison in the first design iteration for the adjoint variable method. In step 1, i.e., in the static analysis, the CPU time for the substructure approach is 197.27 s on an IBM 3090, as opposed to 421.55 s for the full-structure approach. In step 2, which is the problem definition program, the substructure approach has only a small advantage. In step 3, which is the reanalysis for the adjoint loads (artificial loads), the full-structure approach has a small advantage, since the substructure approach includes some overhead for additional loads formulation. In step 4, which is the sensitivity calculation and optimization module, the CPU advantage is 16.88 s to 103.73 s. The savings in this step is primarily due to the integration advantage of the substructure approach in shape sensitivity calculations, in which only the residual structure is integrated. The total CPU time for the substructure approach is 255.9 s, as opposed to 556.65 s for the full-structure approach. The factor of savings is more than 2.

Table 2 shows the CPU time comparison in the first design iteration for the direct method. The CPU advantage is similar to the adjoint variable method. The main difference between Table 1 and 2 is that the integration advantage for the adjoint variable method occurs in step 4, whereas the advantage for the direct method is in step 2, in which the integration for the artificial loads is performed. However, the total factor of savings is also more than 2 in this case.

The optimization results for the substructure and full-structure approach are shown in Fig. 2. From Fig. 2, one sees that the full-structure approach and substructure formulation give very close results. However, the savings in computing resources for the substructure approach is significant (Table 1 and 2). It should be noted that four of the five design iterations are using the adjoint variable method for sensitivity analysis and one is using the direct method.

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